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1993 J. Phys. A: Math. Gen. 26 4419

(http://iopscience.iop.org/0305-4470/26/17/046)

J. Phys. A: Math. Gen. 26 (1993) 4419–4429. Printed in the UK

The eigenvalue spectrum of a large symmetric random matrix with exponential distributed elements

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Abstract. A replica solution for the averaged eigenvalue spectrum of a large symmetric $N \times N$ matrix with an exponential distribution $p(M_{ij}) = (\sqrt{N}/M_{0ij})\exp\{-\sqrt{N}M_{ij}/M_{0ij}\}$ ($M_{0ij} = M_0$) of the elements is presented. This problem is reduced to the solution for the averaged eigenvalue spectrum of a homogeneous matrix $M_{0ij} = M_0$ with an added Gaussian random matrix. The main part of the obtained spectral density is the well known semicircular law $\rho(\lambda) = (1/2\pi M_0^2)\sqrt{4M_0^2 - \lambda^2}$. In contrast to the Gaussian random matrix an additional second spectral region in the vicinity of $M_0\sqrt{N}$ is observed. The analytic result is verified by numerically obtained spectra of such matrices.

1. Introduction

The averaged eigenvalue spectrum of a large $N \times N$ symmetric random matrix each of whose elements has a Gaussian probability distribution of the form

$$p(\{M_{ij}\}) = \prod_{i\leqslant j}\left(\frac{N}{2\pi J^2}\right)\exp\left\{-\frac{N}{2J^2}M_{ij}^2\right\}. \tag{1}$$

is well understood. Edwards and Jones [1] gave a replica solution for this problem which yields the so-called semicircular law [2]. The physical background of this problem was elucidated by Kosterlitz *et al* [3].

The problem of what happens to the density of states $\sigma(\epsilon)$ of a given matrix when disorder of the above type is added was investigated by Edwards and Warner [4] using the replica trick.

In this paper the average eigenvalue spectrum of a large symmetric random matrix each of whose elements has an exponential distribution of the form

$$p(\{M_{ij}\}) = \prod_{i\leqslant j}\frac{\sqrt{N}}{M_{0ij}}\exp\left\{-\sqrt{N}\frac{M_{ij}}{M_{0ij}}\right\} \qquad i, j = 1, \ldots, N \tag{2}$$

is studied. $M_{0ij} = \text{constant} = M_0$ that is equivalent to an unstructured matrix **M**. The factor $\sqrt{N}$ is introduced to be in agreement with [1,4] and is discussed below.

Exponential distributions are very common in statistical physics and are sometimes more natural than Gaussian distributions. Frozen exponential disorder appears whenever gaps of random point processes are of interest. An example is the stochastic crosslink process of a polymer melt. If the *a priori* probability p_0 for a crosslink between two monomers is independent of other monomers and crosslinks, the probability to obtain N_c connected monomers, that is a gap between the crosslink points of length N_c, is given by [5, 6]

$$p(N_c) = p_0(1-p_0)^{N_c} \simeq p_0\, e^{-p_0 N_c}. \tag{3}$$

N_c is the length of the corresponding network chain after crosslinking.

A further example is the random hierarchical distribution of energy barriers discussed by Teitel [7, 8].

However, the mathematical formalism to solve the problem of the random symmetric matrix with an exponential distribution of the elements is somewhat difficult and may serve as a further illustration for the usefulness of the replica method.

2. Replica solution of the averaged eigenvalue spectra

One has to evaluate the expression

$$[\rho(\lambda)] = \int P(\mathbf{M}, \mathbf{M}_0)\rho(\lambda, \mathbf{M})\, d\mathbf{M} \tag{4}$$

for the eigenvalue spectrum averaged over a sample of independent realizations of $\mathbf{M}$. Self-averaging is usually assumed for large matrices ($N \to \infty$). A real symmetric matrix $\mathbf{M}$ with eigenvalues λ_k will have a spectral density

$$\rho(\lambda, \mathbf{M}) = \frac{1}{N}\sum_{k=1}^{N}\delta(\lambda - \lambda_k). \tag{5}$$

It was shown by [1] that the identity

$$\rho(\lambda, \mathbf{M}) = -\frac{2}{N\pi}\,\mathrm{Im}\,\frac{\partial}{\partial\lambda}\ln\det{}^{-1/2}(\tilde{\lambda} - \mathbf{M}) \tag{6}$$

holds with

$$\tilde{\lambda} = \lim_{\epsilon\to 0}(\lambda + i\epsilon). \tag{7}$$

The determinant can be represented by an appropriate Gaussian integration

$$\rho(\lambda, \mathbf{M}) = -\frac{2}{N\pi}\,\mathrm{Im}\,\frac{\partial}{\partial\lambda}\ln\left(\pi^{N/2}\int\{d\phi\}\, e^{-\phi_k(\tilde{\lambda}-\mathbf{M})_{kl}\phi_l}\right). \tag{8}$$

To compute the right-hand side of equation (4) one has to average the logarithm of a Gaussian integral. This can be circumvented using the replica trick [9].

An exact representation of the logarithmic function is

$$\ln(x) = \lim_{n\to 0}\frac{1}{n}(x^n - 1). \tag{9}$$

The crux is to assume n to be a positive integer to take advantage of the vectorization of the integral. The structure average over the matrix $\mathbf{M}$ has then to be taken first. Whenever possible an analytical continuation ($n \to 0$) should be done.

In this way we get

$$[\rho(\lambda, \mathbf{M}_0] = -\lim_{n\to 0} \frac{\partial}{\partial\lambda}\frac{2}{\pi}\,\mathrm{Im}\,\frac{1}{Nn}\{\mathrm{I}(\tilde{\lambda}, \mathbf{M}_0; n) - 1\} \tag{10}$$

with

$$\mathrm{I}(\tilde{\lambda}, \mathbf{M}_0; n) = \pi^{-N.n/2}\int\{\mathrm{d}\phi\}\int \exp\left\{-\sum_{kl}\phi_k^\alpha(\tilde{\lambda} - \mathbf{M})_{kl}\phi_j^\alpha\right\}$$
$$\times\prod_{i\leqslant j}\mathrm{d}M_{ij}\frac{1}{M_{0ij}}\sqrt{N}\exp\left\{-\frac{M_{ij}}{M_{0ij}}\sqrt{N}\right\} \tag{11}$$

where the replica indices α are integers in the range from 1 to n. The double appearance of replica indices should indicate summation over the given range. The integration over the symmetric independent matrix elements yields

$$\int\prod_{i\leqslant j}\mathrm{d}M_{ij}\frac{\sqrt{N}}{M_{0ij}}\exp\left\{-\sqrt{N}\frac{M_{ij}}{M_{0ij}} + \sum_{kl}M_{kl}\phi_k^\alpha\phi_l^\alpha\right\}$$
$$= \prod_{i<j}\frac{1}{1 - 2(M_{0ij}/\sqrt{N})\phi_i^\alpha\phi_j^\alpha}\prod_k\frac{1}{1 - (M_{0kk}/\sqrt{N})\phi_k^\alpha\phi_k^\alpha}. \tag{12}$$

It should be noted that the algebraic expression in equation (12) is just the main problem appearing with the exponential distribution. In the case of a Gaussian distribution the result is a fourth-order term of ϕ in the exponential of $I(\tilde{\lambda}, J; n)$ of the form

$$\exp\left\{\frac{J^2}{N}\sum_{ij}\phi_i^\alpha\phi_j^\alpha\phi_i^\beta\phi_j^\beta - \frac{1}{2}\frac{J^2}{N}\sum_i\phi_i^\alpha\phi_i^\alpha\phi_i^\beta\phi_i^\beta\right\} \tag{13}$$

which will be further evaluated by introducing an auxiliary field [1, 4, 10].

Hence, the result of equation (12) is, at first glance, somewhat disencouraging. However, a solution is possible by recalling that the expression $\phi^\alpha\phi^\alpha$ is of order n. So it may be useful to expand equation (12) in powers of $(M_{0ij}/\sqrt{N})\phi_i^\alpha\phi_j^\alpha$.

$$\prod_{i<j}\frac{1}{1-2\Phi_{ij}}\prod_k\frac{1}{1-\Phi_{kk}}$$
$$\simeq \prod_{i<j}(1 + 2\Phi_{ij} + 4\Phi_{ij}\Phi_{ij} + \cdots)\prod_k(1 + \Phi_{kk} + \Phi_{kk}\Phi_{kk} + \cdots) \tag{14}$$

with

$$\Phi_{ij} \stackrel{\mathrm{def}}{=} \phi_i^\alpha\phi_j^\alpha M_{0ij}/\sqrt{N}$$

and further

$$(14) \simeq \left(1 + \sum_{i\neq j}\Phi_{ij} + 2\sum_{i\neq j}\Phi_{ij}\Phi_{ij} + \frac{1}{2}\sum_{(i\neq j)\neq(k\neq l)}\Phi_{ij}\Phi_{kl} + \cdots\right)$$
$$\times\left(1 + \sum_{i=j}\Phi_{ij} + 2\sum_{i=j}\Phi_{ij}\Phi_{ij} + \frac{1}{2}\sum_{(i=j)\neq(k=l)}\Phi_{ij}\Phi_{kl} + \cdots\right). \tag{15}$$

By evaluating the remaining product and regrouping the summations we get

$$\prod_{i<j}\frac{1}{1-2\Phi_{ij}}\prod_{k}\frac{1}{1-\Phi_{kk}} \simeq 1+\sum_{ij}\Phi_{ij}+\frac{1}{2}\sum_{ij}\sum_{kl}\Phi_{ij}\Phi_{kl}+\cdots$$
$$+\sum_{ij}\Phi_{ij}\Phi_{ij}-\frac{1}{2}\sum_{i}\Phi_{ii}\Phi_{ii}+\cdots. \tag{16}$$

Equation (16) is nothing other than the leading orders of an expansion of

$$\exp\left\{+\sum_{ij}\frac{1}{\sqrt{N}}\phi_i^\alpha M_{0ij}\phi_j^\alpha+\frac{1}{N}\sum_{ij}M_{0ij}M_{0ij}\phi_i^\alpha\phi_j^\alpha\phi_i^\beta\phi_j^\beta\right.$$
$$\left.-\frac{1}{2N}\sum_{i}M_{0ii}M_{0ii}\phi_i^\alpha\phi_i^\alpha\phi_i^\beta\phi_i^\beta\right\}. \tag{17}$$

Hence, only the fourth-order expression in equation (13) is reproduced, with an added quadratic contribution.

From now on M_{0ij} = constant = M_0 has to be taken. By introducing a Gaussian matrix M_{ij} the integral $I(\tilde{\lambda}, \mathbf{M}_0; n)$ is reduced to

$$I(\tilde{\lambda}, M_0; n)=\pi^{-N.n/2}\int\{\mathrm{d}\phi\}\int\prod_{i\leqslant j}\mathrm{d}M_{ij}\exp\left\{-\phi_i^\alpha\left(\tilde{\lambda}-\frac{1}{\sqrt{N}}\mathbf{M}_0-\mathbf{M}\right)_{ij}\phi_j^\alpha\right\}$$
$$\times\left(\frac{N}{2\pi M_0^2}\right)^{1/2}\exp\left\{-\frac{N}{2M_0^2}M_{ij}^2\right\}. \tag{18}$$

By going back over the steps from equation (8) to equation (5) we recover the expression for the averaged eigenvalue spectrum of a matrix M_0 disturbed by a random symmetric matrix M_{ij} each of whose elements has a Gaussian distribution

$$[\rho(\lambda, M_0)]=\int\frac{1}{N}\mathrm{Tr}(\lambda-\mathbf{M})\prod_{i\leqslant j}\mathrm{d}M_{ij}\frac{1}{M_0}\sqrt{N}\exp\left\{-\frac{M_{ij}}{M_0}\sqrt{N}\right\}$$
$$=\int\frac{1}{N}\mathrm{Tr}(\lambda-\mathbf{M}_0-\mathbf{M})\prod_{i\leqslant j}\mathrm{d}M_{ij}\left(\frac{N}{2\pi M_0^2}\right)^{1/2}\exp\left\{-\frac{N}{2M_0^2}M_{ij}^2\right\}. \tag{19}$$

Equation (19) is the main result of this paper.

It can be shown that the orthogonal transformation $\mathbf{O}$ that diagonalized $\mathbf{M}_0$ within the trace operation in equation (19) can be done with no influence on the distribution of $\mathbf{O}^T\mathbf{M}\mathbf{O}$ [4]. Hence, equation (19) can be rewritten as

$$[\rho(\lambda, M_0)]=\int\frac{1}{N}\mathrm{Tr}\left(\left(\lambda-\frac{\lambda_k}{\sqrt{N}}\right)\delta_{kl}-M_{kl}\right)\prod_{i\leqslant j}\mathrm{d}M_{ij}\left(\frac{N}{2\pi M_0^2}\right)^{1/2}\exp\left\{-\frac{N}{2M_0^2}M_{ij}^2\right\}. \tag{20}$$

For the integral $I(\tilde{\lambda}, M_0; n)$ we obtain

$$I(\tilde{\lambda}, M_0; n)=\pi^{-N.n/2}\int\{\mathrm{d}\phi\}\exp\left\{-\sum_{k}\phi_k^\alpha\left(\tilde{\lambda}-\frac{\lambda_k}{\sqrt{N}}\right)\phi_k^\alpha+\frac{1}{N}M_0^2\sum_{ij}\phi_i^\alpha\phi_j^\alpha\phi_i^\beta\phi_j^\beta\right.$$
$$\left.-\frac{1}{2}\frac{M_0^2}{N}\sum_{i}\phi_i^\alpha\phi_i^\alpha\phi_i^\beta\phi_i^\beta\right\}. \tag{21}$$

A Gaussian auxiliary field q^α must be introduced to reduce the fourth-order term in ϕ. The 'devectorization' is obtained by the analytic continuation $n \to 0$. Now the q integration can be carried out using the saddle-point method. For details see [1, 4]. If $g(\lambda, q, \lambda_k)$ denotes the exponent of $I(\cdot)$, the main contribution of the remaining q-integral results from

$$\frac{\partial}{\partial q} g(\lambda, q) = \frac{q}{M_0^2} + \frac{1}{N} \sum_k \frac{1}{\tilde{\lambda} - (\lambda_k/\sqrt{N}) + q} = 0. \tag{22}$$

The averaged eigenvalue density is then given by

$$[\rho(\lambda, M_0)] = \frac{2}{\pi} \operatorname{Im} \frac{\mathrm{d}}{\mathrm{d}\lambda} g(q_0) \tag{23}$$

where q_0 is the solution of equation (22). By using equation (22) we get from equation (23) [4]:

$$[\rho(\lambda, M_0)] = \frac{1}{\pi M_0^2} \operatorname{Im}(q_0(\lambda)). \tag{24}$$

The only problem left is to calculate the imaginary part of the solution of equation (22). The eigenvalues of the matrix $\mathbf{M}_0$ are easy to obtain, see also for example [1]:

$$\lambda_{1,\ldots,N-1} = 0 \tag{25}$$

$$\lambda_N = N \cdot M_0. \tag{26}$$

We now assume that the solution of the spectra exhibits no overlap between the parts arising from the different eigenvalues of $\mathbf{M}_0$. This means that equation (22) may be separated into two independent solutions for both the different eigenvalues. For the zero eigenvalues it yields

$$\frac{q_0}{M_0^2} + \frac{1}{\tilde{\lambda} + q_0} = 0 \tag{27}$$

with

$$(N-1)/N \simeq 1 \qquad (N \gg 1). \tag{28}$$

Hence

$$\operatorname{Im}(q_0(\lambda)) = \tfrac{1}{2}(4M_0^2 - \lambda^2)^{1/2}. \tag{29}$$

The small imaginary part of $\tilde{\lambda}$ can, in this case, be neglected. We therefore reproduce the semicircular law of the form

$$[\rho(\lambda, M_0)] = \frac{1}{2\pi M_0^2}(4M_0^2 - \lambda^2)^{1/2} \qquad (N \gg 1). \tag{30}$$

A second solution appears for $\lambda_N = N \cdot M_0$. In this case the saddle-point method is not satisfactory because of the weight $1/N$ in this contribution of equation (22). In this case it will be better to take advantage of the identity

$$[\rho(\lambda, M_0)] = \operatorname{Im} \frac{2}{N\pi} \frac{1}{n} \sum_{\alpha,k} \langle \phi_k^\alpha \phi_k^\alpha \rangle \tag{31}$$

where the brackets $\langle\rangle$ denote the average with the integral $I(\tilde{\lambda}, M_0; n)$. Equation (31) follows directly after carrying out the differentiation in equation (10) with respect to λ. For the single eigenvalue λ_N we get

$$[\rho_{\lambda_N}(\lambda, M_0)] = \frac{1}{N}\left(\frac{N}{4\pi M_0^2}\right)^{1/2} \exp\left\{-\frac{N}{4M_0^2}(\lambda - M_0\sqrt{N})^2\right\}. \tag{32}$$

In this derivation all correlations of different eigenvalues are neglected. These correlations give rise to a remarkable broadening in the density for $\lambda_{1\ldots N-1}$. But for isolated eigenvalue this is the correct solution, i.e. without saddle-point approximation. Altogether we have

$$[\rho(\lambda, M_0)] = \frac{1}{2\pi M_0^2}(4M_0^2 - \lambda^2)^{1/2} + \frac{1}{N}\left(\frac{N}{4\pi M_0^2}\right)^{1/2} \exp\left\{-\frac{N}{4M_0^2}(\lambda - M_0\sqrt{N})^2\right\}. \tag{33}$$

This is correct normalized up to the order $1/N$ because of equation (28). This slight overestimation of the main spectral part should be of no interest here.

The following points should be noted in conclusion.

(i) The surprising equivalence of the main part of the eigenvalue density (equation (30)) with the corresponding solution for the Gaussian case. Note that for the exponential distribution only positive matrix elements occur in contrast to the case of a Gaussian distribution.

(ii) The appearance of a second spectral region far above the first one ($M_0\sqrt{N} \gg 2M_0 \Rightarrow \sqrt{N} \gg 2$) which is the only difference to the Gaussian case.

(iii) The $\sqrt{N}$ in the exponential distribution (equation (2)) ensures the validity of the saddle-point method. Nevertheless the solution in equation (33) also holds for much larger fluctuations, which will be shown in the next section.

3. Comparison of the numerically obtained spectral densities and the analytical results

To check the analytic results eigenvalue spectra of matrices with a dimensionality in the range from 50 to 200 were numerically computed. M_0 was first set to unity, as demanded by the saddle-point approximation. A second set of matrices with $M_0 = 750.0$ was diagonalized to study the spectra far beyond the range of the approximation. The spectral density of the corresponding Gaussian matrices was also computed. To obtain satisfactory data for the eigenvalue spectra 10^4 matrices were used for each parameter set. The numerical work was done on the CRAY Y-MP of the University of Regensburg using NAG routines for the diagonalization.

In figure 1 the eigenvalue spectra of exponential matrices for different choices of N with $M_0^2 = 1.0$ are displayed. The scaling in the main spectral region around $\lambda = 0.0$ is striking, see also figure 3. The second spectral region scales according to equation (32). The case $M_0^2 = 750.0$ is plotted in figure 2. In figure 4 the numerical results of the eigenvalue spectra for the exponential and the Gaussian distribution are compared with the analytical result of equation (30) for $N = 50$. The deviation in the case of the exponential distribution is somewhat larger than for the Gaussian matrices. Nevertheless there is satisfactory agreement between the analytical and numerical results. Note that even in the Gaussian case there are small deviations from the theoretical predictions. These deviations cannot arise from the saddle-point method because there is no remarkable difference between $M_0^2 = 1.0$

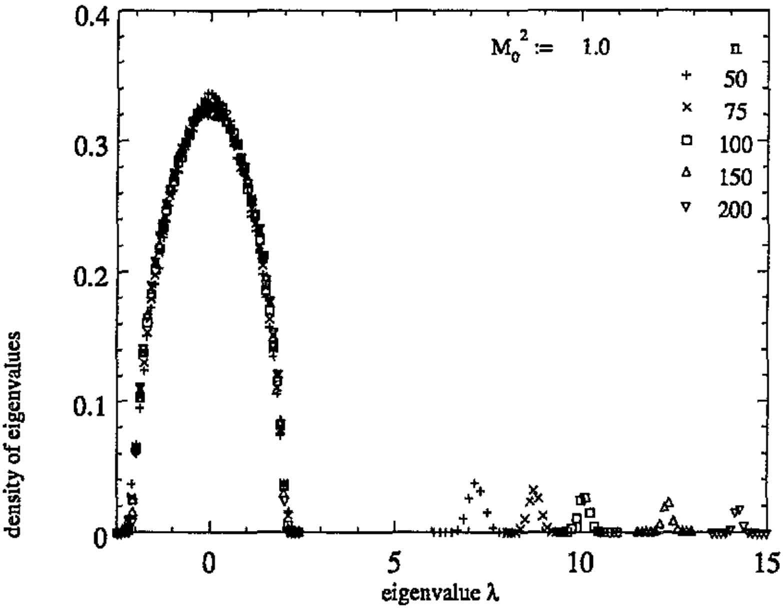

Figure 1. Spectral density for matrices with an exponential distribution of elements ($M_0^2 = 1.0$).

and $M_0^2 = 750.0$. The other approximation was to neglect the non-symmetric replica contribution [10]. Edwards and Warner [4] argued with a perturbation calculation. However, in the case of the exponential distribution, the replica symmetric approximation is more difficult due to neglect of higher-order terms in the expansion of the product in equations (14) and (15).

In figures 5 and 6 the second part of the spectrum is compared with equation (32) for $N = 50$ and both values of M_0^2. A shift of about 1.0% between the analytical and the numerical result is observed. However, the width as well as the height are as predicted. This is by no means trivial, because the original distribution is exponential rather than Gaussian. The spectrum of a Gaussian matrix exhibits no such second spectral region at all.

The shift can be understood by means of a direct pertubation calculation. To do so it is useful to divide the matrix **M** into two parts, i.e.

$$M_{ik} = \langle M_{ik} \rangle + \delta M_{ik} \tag{34}$$

where the first part results in a simple homogeneous matrix. In this case the eigenvalues are: $\lambda_{1,\ldots,N-1} = 0$ and $\lambda_N = \sqrt{N} \cdot M_0$. The difference to equation (25) arises due to the factor $\sqrt{N}$ in the distribution function, see equation (2). Our aim is to calculate the mean of the eigenvalue λ_N by means of the simple Schrödinger pertubation method commonly used for quantum mechanical eigenvalue problems. The first order of the pertubation series vanishes because $\langle \delta M_{ik} \rangle = 0$, which follows clearly from equation (34). For the correction of λ_N in second order one obtains

$$\lambda_N^{(2)} = \left\langle \sum_{i<N} \frac{\langle m_N | \delta\mathbf{M} | m_i \rangle \langle m_i | \delta\mathbf{M} | m_N \rangle}{\lambda_N} \right\rangle. \tag{35}$$

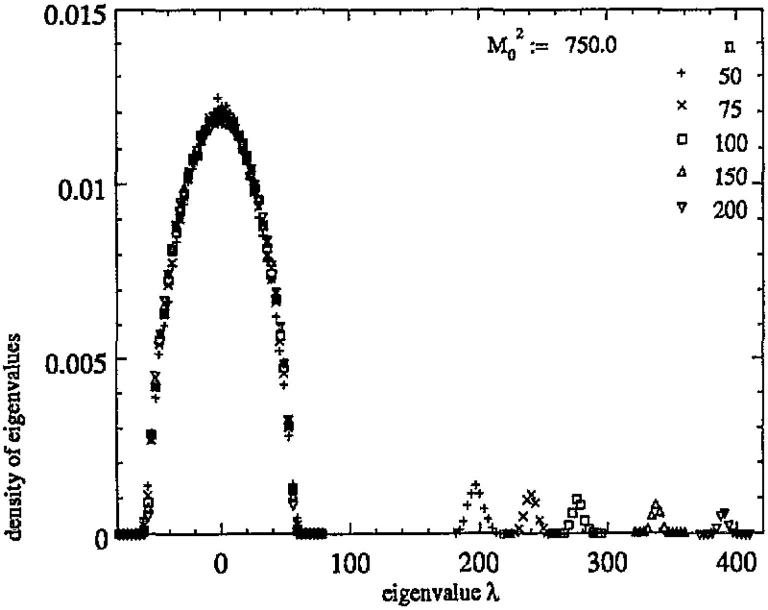

Figure 2. Spectral density for matrices with exponential distribution of elements ($M_0^2 = 750.0$).

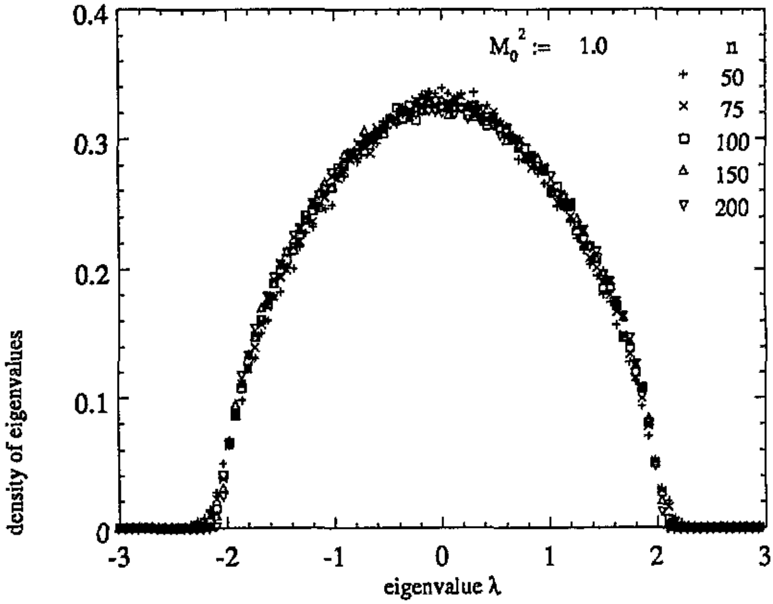

Figure 3. Numerically obtained spectral density of matrices with exponential distribution of elements in the main spectral region ($M_0^2 = 1$).

λ_N should be the undisturbed eigenvalue. The inner brackets and the bars are chosen in analogy to the quantum mechanical notation, while the outer brackets again denote the

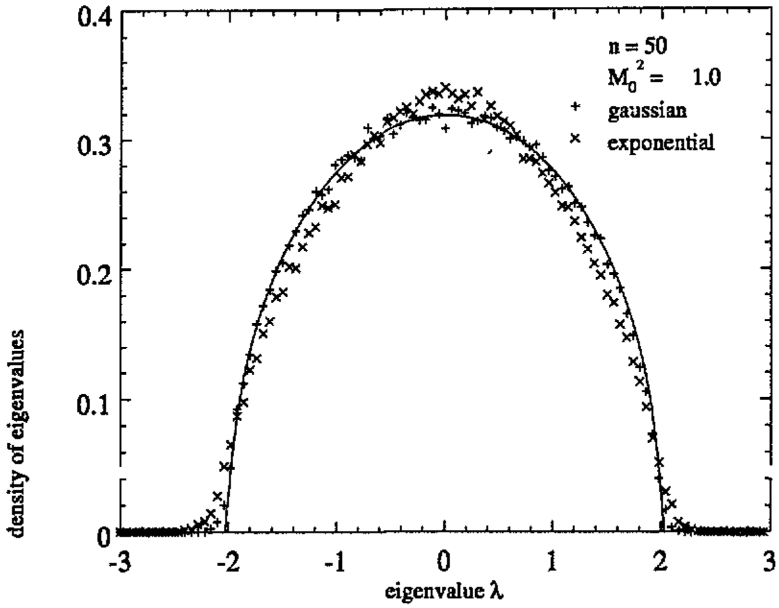

Figure 4. Semicircular law for numerically obtained spectral density for matrices with exponential and Gaussian distribution of elements ($M_0^2 = 1$).

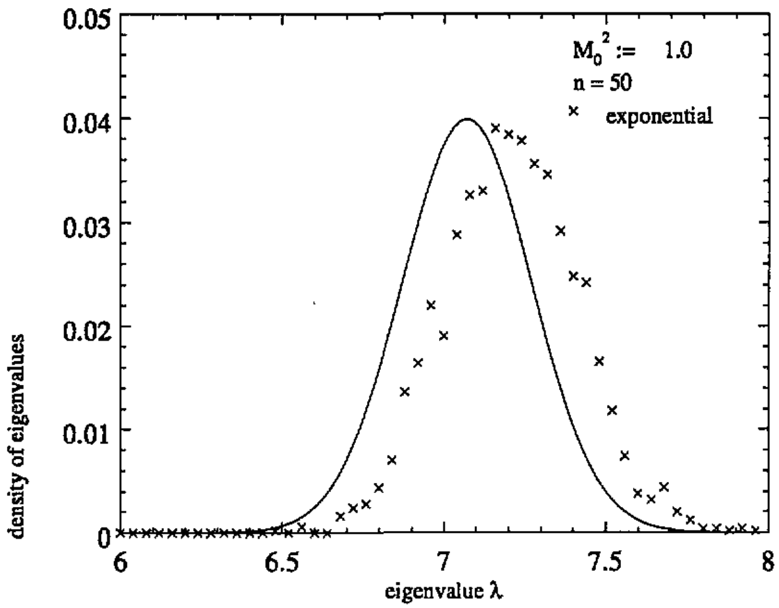

Figure 5. Analytical prediction and numerical results for matrices with an exponential distribution of elements in the second spectral region ($M_0^2 = 1.0$).

average over the matrix ensemble. $|m_{1\ldots N}\rangle$ should be the normalized eigenvectors of the matrix ($\mathbf{M}$). Equation (35) can be rewritten as

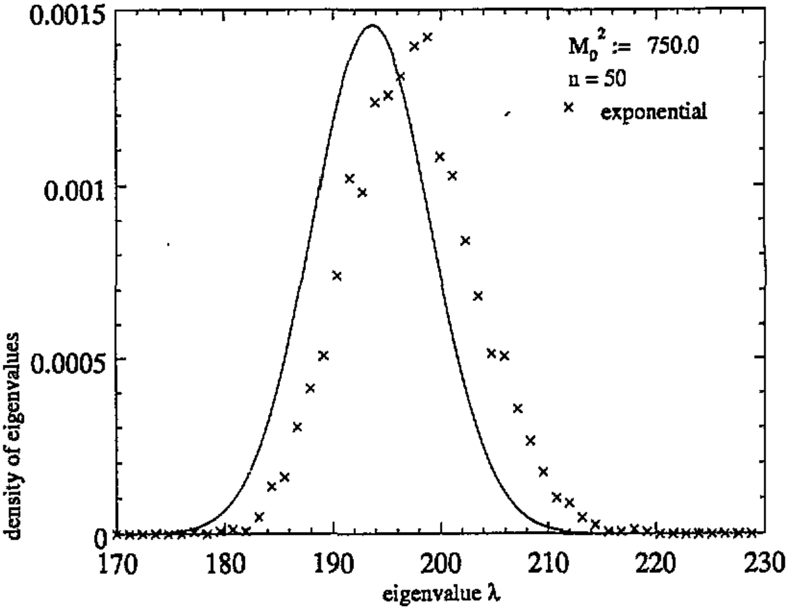

Figure 6. Analytical prediction and numerical results for matrices with an exponential distribution of elements in the second spectral region ($M_0^2 = 750.0$).

$$\lambda_N^{(2)} = \sum_{i<N} \sum_{k,l=1}^{N} \sum_{m,n=1}^{N} m_N^k m_i^l m_i^m m_N^n \langle \delta M_{kl} \delta M_{mn} \rangle. \tag{36}$$

Because of the symmetry of the matrix the statistical average yields

$$\langle \delta M_{kl} \delta M_{mn} \rangle = (\delta_{km}\delta_{ln} + \delta_{kn}\delta_{lm}) M_0^2/N \tag{37}$$

Using the ortho-normalization of the eigenvector set the final result is

$$\lambda_N^{(2)} = \frac{N-1}{N} \frac{M_0}{\sqrt{N}}. \tag{38}$$

This means a positive shift of the mean position of the single eigenvalue by $M_0/\sqrt{N}$.

Note that the symmetry of the matrix produces a non-vanishing contribution in the second order. Only the second term of equation (37) is responsible for the result in equation (38).

The observed shift in figure 5 is about $\langle \delta\lambda_N \rangle \simeq 0.13$ and the calulation in the second order yields 0.14. For the data in figure 6 we have $\langle \delta\lambda_N \rangle \simeq 3.45$ and the second-order calculation yields 3.87.

In the case of the main spectral region this calculation makes no sense because of the large degeneration in the eigenvalues. Neither is such a shift observable from the numerical data.

In conclusion we can note that the replica solution for the problem of random symmetric matrices with an exponential distribution of the elements given by equations (19) and (33) can be verified by the numerical calculations. It demonstrates once more that this method is very useful for investigating disordered systems even in the case of non-Gaussian disorder.

Acknowledgments

We would like to thank T A Vilgis for useful and stimulating discussions. JUS thanks the Deutsche Forschungsgemeinschaft (DFG) for financial support (So 277/1-1). We are grateful to the Rechenzentrum der Universität Regensburg for providing time and support on the CRAY YM-P.

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